TRINITY COLLEGE

Mathematics Methods 3 & 4
Test 1 2016

Section 1 Calculator Free

Differentiation, Anti-differentiation and their applications.

STUDENT'S NAME ______________________________

DATE: Friday 4th March **TIME:** 25 minutes **MARKS**: 28

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, Formula sheet.

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (6 marks)

Differentiate the following. Do not simplify your answer.

(a) $y = (4x+7)^3(9x-5)$ [3]

(b) $y = \dfrac{\sqrt{2x^5}}{\sqrt{x+7}}$ [3]

2. (3 marks)

Determine $\int 2x(7-3x^2)^4\,dx$

3. (3 marks)

Given that $\int_1^a (2x-3)dx = 6$, determine a.

4. (6 marks)

The air in a hot air balloon is being inflated such that the rate of change of its volume at any time t, minutes, is given as:

$$\frac{dV}{dt} = 3t^2 - 2t \qquad \text{for} \qquad t \geq 0$$

If initially the balloon has 3 m^3 of air in it, determine:

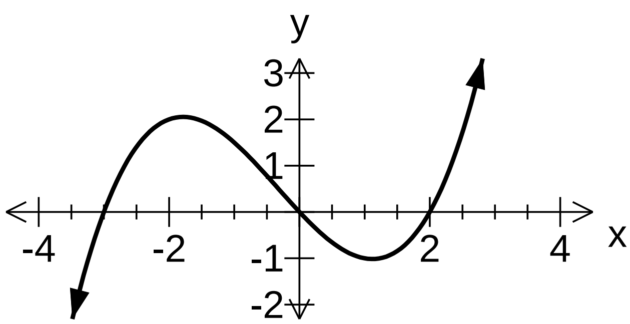
(a) The rate of change in volume when $t = 1$. Explain the meaning of this. [2]

(b) For what values of t the volume is increasing. [2]

(c) The volume of the balloon after five minutes. [2]

5. (4 marks)

The graph of $y = f(x)$ is shown below.



Given $\int_{-3}^{0} f(x)dx = 4$ and $\int_{0}^{2} f(x)dx = -1$, determine the following:

(a) $\int_{-3}^{2} f(x)dx$ [1]

(b) $\int_{0}^{2} 5f(x)dx$ [1]

(c) $\int_{-3}^{2} |f(x)|dx$ [1]

(d) The area enclosed by $f(x)$ and the x axis. [1]

6. (6 marks)

Given the function $y = (x+2)(x^2 - 4x + 4)$.

(a) Determine the gradient of the tangent to the curve at $x = 3$. [3]

(b) Using calculus techniques, determine the nature of the stationary point at $x = 2$. [3]

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Mathematics Methods 3 & 4
Test 1 2016

Section 2 Calculator Assumed
Differentiation, Anti-differentiation and their applications.

STUDENT'S NAME ____

DATE: Friday 4th March **TIME:** 25 mins **MARKS**: 23

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, Formula sheet.

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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7. (6 marks)

The volume Vcm^3 of water in a vessel is given by $V = \frac{1}{6}\pi x^3$, where x cm is the depth of the water in the cylinder in cm.

(a) Determine an approximation for the change in depth when the volume of water changes from 200 to 210 cm^3. [3]

(b) Determine the percentage change in the volume of the vessel if the depth has increased by 6%. [3]

8. (4 marks)

A company manufacturing a new bike determines that the marginal cost (in dollars) for the production of the n^{th} unit is given by the expression:

$$\frac{dC}{dn} = \frac{200000}{(n+20)^2}$$

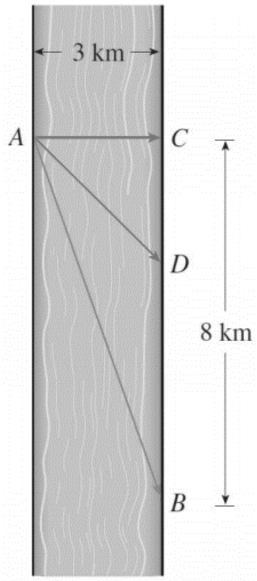
(a) The initial set up cost is \$ 10 000 (i.e. the cost of producing no bikes is \$ 10 000). Show that the expression for the total cost of producing n bikes is:

$$C = \frac{-200000}{n+20} + 20000$$ [2]

(b) If the company sells each bike for \$200, how many bikes must be sold before it first makes a profit? [2]

9. (7 marks)

A man launches his boat from point A on a bank of a straight river, 3 km wide, and wants to reach point B, 8 km downstream on the opposite bank, as quickly as possible.



He could proceed in any of three ways:

1. Row his boat directly across the river to point C and then run to B
2. Row directly to B
3. Row to some point D between C and B and then run to B

The man can row at a speed of 6 km/h and run at a speed of 8 km/h.

(a) Given that $time = \frac{distance}{speed}$ and x is the distance from C to D, show that the time (t) taken for the man to travel from A to B can be represented by the equation. [2]

$$t = \frac{\sqrt{x^2+9}}{6} + \frac{8-x}{8}$$

(b) Using calculus techniques, determine the minimum time taken by the man to reach point B and the distance he would travel by foot to achieve this minimum time. [5]

10. (6 marks)

A particle moves in rectilinear motion with a velocity of 7 m/s as it passes through a fixed point O.

t is the number of seconds since passing through O. Acceleration a is defined as $a = mt - n$, where m and n are constants.

When $t = 1$, the velocity is 12 m/s, and when $t = 7$ the particle is instantaneously at rest.

(a) Calculate the values of m and n. [3]

(b) Hence, determine the expression for the velocity as a function of time. [1]

(c) Determine when and where the maximum velocity is attained. [2]